The number of different coefficients in the expanded form is much less than the number of terms. In the ninth power of nine quantities there are 24310 terms, but only 30 different coefficients.

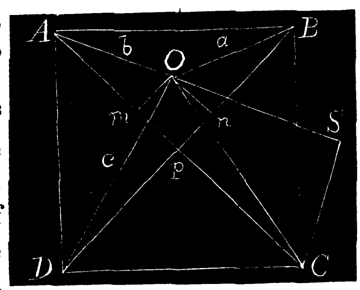
The above demonstration becomes imaginary when n is fractional or negative.

ANSWER TO QUERY AT P. 64, VOL. IV, BY PROF. KERSHNER.— "The extraction of the square root of $a + b\sqrt{-1}$ is an operation to which Euclid's geometry is competent; it requires only the bisection of an angle and the determination of a mean proportional to obtain

$$\left\{ \sqrt[4]{(a^2+b^2)},\ \tfrac{1}{2}\tan^{-1}\frac{b}{a} \right\} \text{ from } \left\{ \sqrt{(a^2+b^2)},\ \tan^{-1}\frac{b}{a} \right\}.$$

Hence it follows that wherever n is a prime number, and $n-1$ is a power of 2, the formation of the nth roots of unity is a geometrical operation, in the ancient sense. This is the discovery of Gauss, and is the most remarkable addition to the power of construction which the ancient geometry has received since the time of Euclid. Euclid mastered the cases $n=3$, $n=5$; the next one is $n=17$, and the next $n=257$."—See De Morgan's *Trigonometry and Double Algebra.*

GEOMETRICAL SOLUTION OF PROB. 125, BY PROF. W. P. CASEY.—AO, BO and DO are given to find the side of the square.



Join AC, BD and CO and draw Om, On, perpendiculars, and also Cs perpendicular to AO produced.

It is well known that $AO^2 + CO^2 = DO^2 + BO^2$, i. e., the sum of the squares of the lines drawn from any point to the opposite angles of a square or rectangle = the sum of the squares of the two lines drawn from the same point to the other two opposite angles, and therefore $AO^2 + CO^2$ is given, and AO is given, $\therefore$ CO is a given line.

Again, $DO^2 - BO^2 = Dn^2 - Bn^2 = 2BD \times pn = 2AC \times Om$. But $2AC \times Om = 2AO \times Cs$, hence $DO^2 - BO^2 = 2AO \times Cs$; $\therefore$ $2AO \times Cs$ is given and AO is given, $\therefore$ Cs is given; and CO is given, by the above, $\therefore$ $CO^2 - Cs^2 = Os^2$ is given, and $\therefore$ Os is a given line and AO is given, hence As is given and so is Cs, hence AC is a given line, and $\therefore$ $AC^2 = 2AB^2$ is given and hence AB is a given line.

[The above solution is here given because it is purely geometrical, whereas the solution published at p. 188, Vol. III, contains trigonometric funct's.]